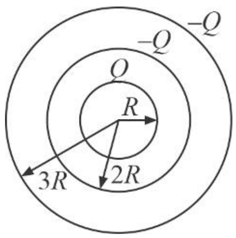
Solutions to JEE Advanced Booster Test – 1|JEE 2024

PHYSICS

SECTION-1

1.(B)



$$\frac{Q^2}{8\pi\varepsilon_0 R}+\frac{(-Q)^2}{8\pi\varepsilon_0(2R)}+\frac{(-Q)^2}{8\pi\varepsilon_0(3R)}+\frac{-Q^2}{4\pi\varepsilon_0(2R)}-\frac{-Q^2}{4\pi\varepsilon_0(3R)}+\frac{Q^2}{4\pi\varepsilon_0(3R)}=\frac{5Q^2}{48\pi\varepsilon_0 R}$$

2.(A) $E=\dfrac{2K_p}{\left(r-\dfrac{a}{2}\right)^3}-\dfrac{2K_p}{\left(r+\dfrac{a}{2}\right)^3}$

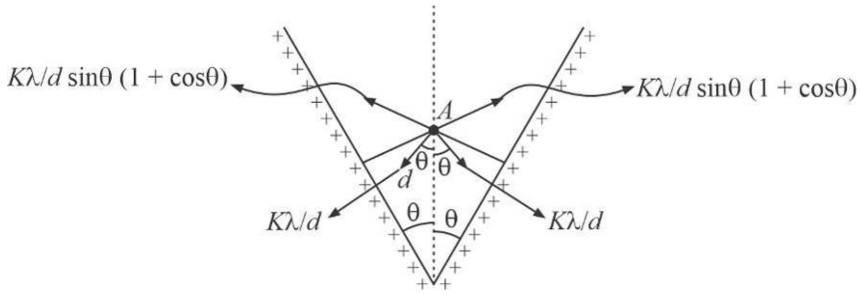
$$\Rightarrow \quad E=2K_p\left[\frac{1}{\left(r-\frac{a}{2}\right)^3}-\frac{1}{\left(r+\frac{a}{2}\right)^3}\right]$$

$$\Rightarrow \quad E=2K_p\left[\frac{\left(r+\frac{a}{2}\right)^3-\left(r-\frac{a}{2}\right)^3}{\left[r^2-\frac{a^2}{4}\right]^3}\right]$$

$$\Rightarrow \quad E=2K_p\left[\frac{3r^2a+\frac{a^3}{4}}{\left(r^2-\frac{a^2}{4}\right)^3}\right]$$

$$\Rightarrow \quad E=\frac{6K_pr^2a}{r^6} \quad \Rightarrow \quad E=\frac{6K_pa}{r^4} \quad \Rightarrow \quad E=\frac{6ap}{4\pi\varepsilon_0 r^4}$$

3.(B)



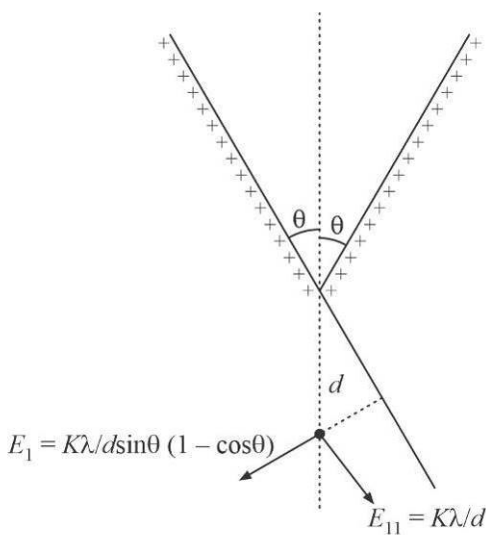
$$E_{\text{net}_1} = \frac{2K\lambda}{d}(1+\cos\theta) - \frac{2K\lambda}{d}(\cos\theta) = \frac{2K\lambda}{d}$$

$$E_{\|_{net}} = \frac{2K\lambda}{d}\cos\theta$$

$$E_{\perp_{net}} = \frac{2K\lambda}{d}(1-\cos\theta)$$

$$E_{\text{net}_2} = \frac{2K\lambda}{d}$$

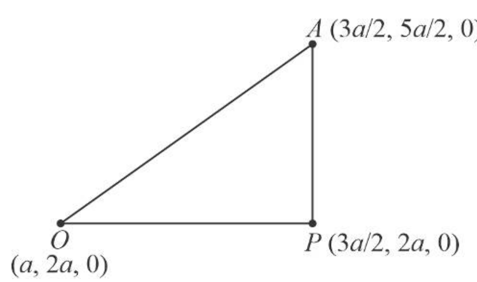
$$\text{Ratio} = \frac{E_{\text{net}_1}}{E_{\text{net}_2}} = \frac{1}{1}$$



4.(C) If the charge is placed at the centre of a cube of side 'a', electric flux through the cube will be $\phi_{\text{cube}} = \frac{q}{\varepsilon_0}$.

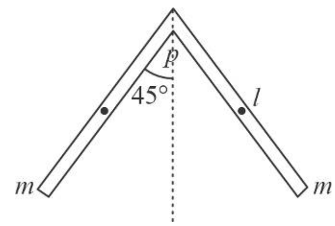
Flux through any square face would be $\phi = \frac{q}{6\varepsilon_0}$.

Imagine the charge to be above vertex O of the triangle OAP.



Area of ΔOAP is $1/8^{th}$ of square face of side 'a' at depth $a/2$ below the charge. Hence flux through this triangle $=\frac{1}{8}\left(\frac{q}{6\varepsilon_0}\right)=\frac{q}{48\varepsilon_0}$

5.(B) $T=2\pi\sqrt{\frac{I}{MgL}}$



$$\Rightarrow \quad T=2\pi\sqrt{\frac{ml^2/3+ml^2/3}{\frac{2mgl}{2}\times\frac{1}{\sqrt{2}}}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{\frac{2ml^2}{3}}{\frac{2mgl}{2\sqrt{2}}}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$$

6.(ABCD)

(A) $T=2\pi\sqrt{\frac{6m}{K}}$

(B) Equilibrium compression $=\frac{6mg}{K}$

Maximum compression $=\frac{12mg}{K}$ $\quad\Rightarrow\quad$ Amplitude $A=\frac{6mg}{k}$

(C) Maximum compression $=\frac{6mg}{K}$

(D) For compression $=\frac{8mg}{K}$ displacement of block from mean position is

$$x=\frac{8mg}{K}-\frac{6mg}{K}=\frac{2mg}{K}$$

$$\Rightarrow \quad V\sqrt{\frac{K}{6m}}\sqrt{\left(\frac{6mg}{K}\right)^2-\left(\frac{2mg}{K}\right)^2}=\sqrt{\frac{K}{6m}}\sqrt{\frac{32(mg)^2}{K^2}}$$

$$\Rightarrow \quad V=\sqrt{\frac{16mg}{3K}} \qquad \Rightarrow \quad V=\sqrt{\frac{16mg}{3K}}$$

7.(AD) Potential of A = Potential of B

$V_A=V_B$

$$\Rightarrow \quad \frac{Q-q}{4\pi\varepsilon_0 R}=\frac{q}{4\pi\varepsilon_0(2R)}+\frac{(-q)}{4\pi\varepsilon_0\left(\frac{21R}{10}\right)}$$

$$\Rightarrow \quad Q - q = \frac{q}{2} - \frac{10q}{21}$$

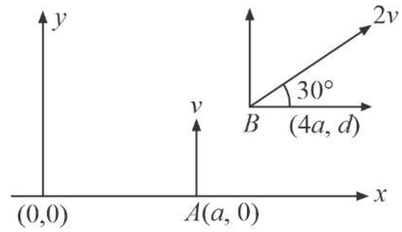
$$\Rightarrow \quad Q = \frac{3q}{2} - \frac{10q}{21} = \frac{63q - 20q}{42} = \frac{43q}{42}$$

$$\Rightarrow \quad q = \frac{42Q}{43}, Q - q = \frac{Q}{43}$$

8.(BD)

(B) For electric field along x-axis

$$(2v\cos 30°)^2 = -2\left(\frac{eE_x}{m}\right)(3a)$$



$$4v^2 \times \frac{3}{4} = -\frac{6eaE_x}{m}$$

$$E_x = -\frac{3mv^2}{6ea}, E_x = \frac{mv^2}{2ea}(-\hat{i})$$

For electric field along y-axis

$$\Rightarrow \quad (2v\sin 30°)^2 = v^2 - \frac{2E_y}{m}(d)$$

$$\Rightarrow \quad v^2 = v^2 - \frac{2E_y d}{m} \quad \Rightarrow \quad E_y = 0$$

(D) Rate of doing work at B is

$$P = \vec{F} \cdot \vec{v} = \frac{emv^2}{2ea}\hat{i} \cdot 2v\cos 30°\,\hat{i}$$

$$\Rightarrow \quad P = \frac{\sqrt{3}mv^3}{2a}$$

9.(ACD)

(A) The electric field at O is

$$E_0 = \sqrt{\left(\frac{Kq/4}{R^2}\right)^2 + \left\{\frac{K(3q/4)}{R^2}\right\}^2 + \frac{2K\,q/4}{R^2} \times \frac{K(3q/4)}{R^2} \times \cos 60°}$$

$$\Rightarrow \quad E_0 = \frac{Kq}{R^2}\sqrt{\frac{1}{16} + \frac{9}{16} + 2 \times \frac{3}{16} \times \frac{1}{2}}$$

$$\Rightarrow \quad E_0 = \frac{Kq}{R^2}\sqrt{\frac{13}{16}} \Rightarrow \quad E_0 = \frac{\sqrt{13}q}{16\pi\varepsilon_0 R^2}$$

(B) Potential energy of the system is

$$V = \frac{K(q/2)}{2R\frac{\sqrt{3}}{2}} + \frac{K(-3q/4)(q/4)}{R} + \frac{K(q/2)(q/4)}{2R}$$

$$\Rightarrow \qquad V = -\frac{(\sqrt{3}+1)q^2}{32\pi\varepsilon_0 R}$$

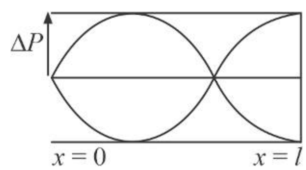
(C) Magnitude of force between B and C is

$$F_{BC} = \frac{\frac{1}{4\pi\varepsilon_0}\left[\frac{3q}{4}\right]\left[\frac{q}{4}\right]}{R^2} = \frac{3q^2}{64\pi\varepsilon_0 R^2}$$

(D) Potential at O is

$$V_0 = \frac{K(-3q/4)}{R} + \frac{K(q/4)}{R} + \frac{K(q/2)}{R} = 0$$

10.(ACD)



$$f = \frac{3v}{4l}$$

$$l = \frac{3v}{4f} = \frac{3\times 330}{4\times 550}$$

$$l = \frac{9}{20} m = 45\ cm$$

The amplitude of pressure variation in the pipe is $a = |\Delta P_0 \sin kx|$

$$a = \left|\Delta P_0 \sin\left(\frac{3\pi}{2l}x\right)\right|$$

At $\qquad x = \frac{l}{2}$

$$a = \left|\Delta P_0 \sin\left(\frac{3\pi}{4}\right)\right| = \frac{\Delta P_0}{\sqrt{2}}$$

The maximum pressure at the middle of the pipe $= \left(P_0 + \frac{\Delta P_0}{\sqrt{2}}\right)$

The maximum pressure at the closed end of the pipe $= (P_0 + \Delta P_0)$

PARAGRAPH FOR Q-11 & 12

11.(B) For Potential at O

$$V_0 = \frac{\rho R^2}{6\in_0}\left[3 - \frac{1}{16}\right] - \frac{\rho\left(\frac{R}{2}\right)^2}{6\in_0}(3) = \frac{35\rho R^2}{96\in_0}$$

12.(A) For electric field at O

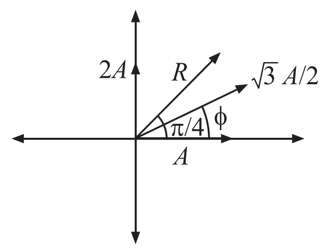
$$E_0 = \frac{\rho(R/4)}{3\varepsilon_0} = \frac{\rho R}{12\varepsilon_0}$$

SECTION-2

1.(3) From the phasor diagram as shown, we can conclude

$$\frac{\sqrt{3}A}{2}\sin\phi + 2A = R\sin\frac{\pi}{4} \qquad \dots (1)$$

$$\frac{\sqrt{3}A}{2}\cos\phi + A = R\cos\frac{\pi}{4} \qquad \dots (2)$$



Phasor diagram

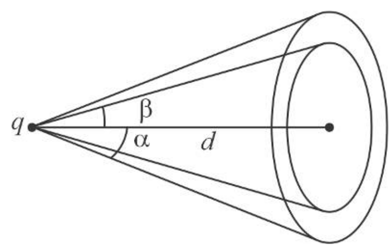
$$\Rightarrow \quad \frac{\frac{\sqrt{3}A}{2}\sin\phi + 2A}{\frac{\sqrt{3}A}{2}\cos\phi + A} = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \frac{\sqrt{3}A}{2}\sin\phi + 2A = \frac{\sqrt{3}A}{2}\cos\phi + A \quad \Rightarrow \quad \cos\left(\phi + \frac{\pi}{4}\right) = \sqrt{\frac{2}{3}} \quad \Rightarrow \quad \phi = 3^\circ$$

2.(5) $\phi = \frac{q}{2\varepsilon_0}(1 - \cos\theta)$

So, flux through the area of the disc between radius 9 *cm* and radius 16 *cm* is

$$\phi = \frac{2q_0}{2\varepsilon_0}[(1-\cos\alpha) - (1-\cos\beta)]$$



$$\Rightarrow \quad \phi = \frac{q_0}{2\varepsilon_0}[\cos\beta - \cos\alpha]$$

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0}\left[\frac{12}{15} - \frac{12}{20}\right]$$

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0}\left[\frac{48-36}{60}\right] \quad \Rightarrow \quad \phi = \frac{q}{\varepsilon_0}\left[\frac{1}{5}\right] \quad \Rightarrow \quad \phi = \frac{q}{5\varepsilon_0} \quad \Rightarrow \quad n = 5$$

3.(1.25)

$$f = \frac{v}{4\ell}, \quad f' = \frac{v}{4\ell'}$$

$$f' = \frac{320}{320-2}f \quad \Rightarrow \quad \frac{f'}{f} = \frac{320}{318}$$

$$\frac{f}{f'} = \frac{\ell'}{\ell} \quad \Rightarrow \quad \frac{318}{320} = \frac{\ell'}{\ell}$$

$$1 - \frac{318}{320} = 1 - \frac{\ell'}{\ell} = \frac{\ell - \ell'}{\ell} = \frac{\Delta\ell}{\ell}; \qquad \frac{\Delta\ell}{\ell} = \frac{2}{320}$$

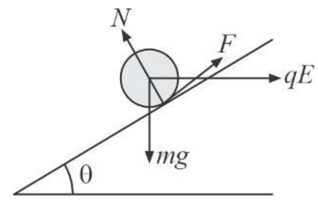
Percentage change $= \frac{2}{320} \times 100 = 0.625$

4.(3) $mg\sin\theta - qE\cos\theta - F = ma \qquad \dots (1)$

$FR = \frac{2}{5}MR^2\alpha \qquad \dots (2)$

$N = mg\cos\theta + qE\sin\theta$ ... (3)

$\therefore \quad a = R\alpha$ ... (4)



From (2) and (4) we get $a = \dfrac{5F}{2m}$ ... (5)

From (1) and (6) get

$mg\sin\theta - qE\cos\theta = \dfrac{7F}{2}$ ... (6)

Now $F = \dfrac{2}{7}(mg\sin\theta - qE\cos\theta)$ ... (6a)

From (3) and 6a we get

$\dfrac{2}{7}(mg\sin\theta - qE\cos\theta) = \mu(mg\cos\theta + qE\sin\theta)$ ... (7)

Using $\mu = \dfrac{1}{7}$ and $\theta = 45°$, we get

$\dfrac{mg}{qE} = 3$

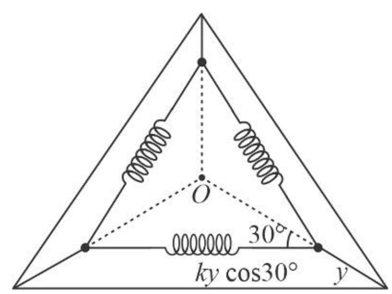
5.(6) Net restoring force $k(2y\cos 30°)\cos 30° \times 2$

$\Rightarrow \quad -4\,ky\cos^2 30° = ma$

$\Rightarrow \quad -4\,ky\cos^2 30° = ma$

$\Rightarrow \quad -4(ky)\times(3/4) = ma$

$\Rightarrow \quad a = -\dfrac{3k}{m}y$



$\Rightarrow \quad \omega = \sqrt{\dfrac{3k}{m}} \quad \Rightarrow \quad \omega = \sqrt{\dfrac{6k}{2m}} \quad \Rightarrow \quad \beta = 6$

6.(2)

$\sqrt{\dfrac{YRT}{M_{H_2}}} = 1270$

Now $\quad PV = nRT$

$\Rightarrow \quad n \propto V$, for same P and T

$\Rightarrow \quad n_{O_2} : n_{H_2} = 1:14$

$\Rightarrow \quad M_{\text{mix}} = \dfrac{n_{O_2}\times M_{O_2} + n_{H_2}\times M_{H_2}}{n_{O_2}+n_{H_2}} \quad \Rightarrow \quad M_{\text{mix}} = \dfrac{1\times 32 + 14\times 2}{1+14} = \dfrac{60}{15} = 4$

$\Rightarrow \quad V_{\text{mix}} = \sqrt{\dfrac{YRT}{M_{\text{mix}}}} = \sqrt{\dfrac{YRT}{4}} = \sqrt{\dfrac{1}{2}}\sqrt{\dfrac{YRT}{2}} = \dfrac{1270}{\sqrt{2}}$

CHEMISTRY

SECTION-1

1.(A) $PV = nRT$

$$\frac{740}{760}\times 15 = n\times 0.0821\times 293$$

$n = 0.607$

$0.607\times\frac{70}{100} = 0.42$ ………………… 15 L of nitrogen

$0.42\times 28 = 11.90$ g

2.(C) There are $4\,C_{60}$ units hence 12 Na, 4 from octahedral and 8 from tetrahedral voids. Hence formula will be $Na_{12}\cdot C_{60\times 4} = Na_3C_{60}$.

3.(A) Volume $a\times b\times c = 215.424\ (\text{Å})^3$

$= 215.424\times 10^{-24}\ cm^3$

$= 2.154\times 10^{-22}\ cm^3$

$$d = \frac{z.M}{N_0a^3} = \frac{4\times 21.76}{6.023\times 10^{23}\times 2.154\times 10^{-22}} = \frac{27.04}{129.4} = 0.67\ g/cc$$

4.(C) Loss in weight of solution $\propto$ V.P. of solution (p_s)

Loss in weight of solvent $\propto p^\circ - p_s$

$$\therefore \quad \frac{p^\circ - p_s}{p_s} = \frac{\text{Loss in weight of solvent}}{\text{Loss in weight of solution}} = \frac{w_2/M_2}{w_1/M_1}$$

$$\frac{0.05}{2.5} = \frac{10/M_2}{90/18} = \frac{2}{M_2}$$

or $\quad M_2 = 100\ g/mol$

5.(C) For two immiscible liquid

$P_A^\circ = P_{total} - P_{H_2O}^\circ = 748 - 648 \Rightarrow 100$

$$\frac{W_A}{W_B} = \frac{P_A^\circ M_A}{P_B^\circ M_B},\ M_A = \frac{1.25}{1}\times\frac{648\times 18}{100}$$

$\Rightarrow \quad 145.8$

6.(ABC) Conceptual

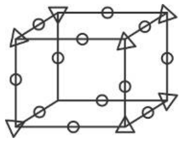
7.(ABCD)

(A) $\quad$ Mn ion $= 8\times\frac{1}{8} = 1$

F^- ion $= 12\times\frac{1}{4} = 3$

Empirical formula $= MnF_3$

(B)



$\Delta -$ Mn ion

$O - F^-$ ion

Coordination number of Mn ion = 6

(C) Coordination number of F^- ion = 2

(D) $r_{Mn^{3+}} + r_{F^-} = \frac{a}{2}$

$a = 2(0.65 + 1.36)$

$a = 2(2.01) = 4.02$ Å

8.(ACD)

Average velocity $= \frac{V_1 + V_2 + V_3 + \ldots V_n}{N} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi M}} = \sqrt{\frac{8P}{\pi d}}$

9.(AB) For the formation of an ideal solution:

$\Delta G = -ve, \Delta S_{system} = +ve, \Delta S_{surrounding} = 0, \Delta H = 0$

10.(CD) Conceptual

PARAGRAPH FOR Q-11 & 12

11.(B) $d_{z^2} + p_y \rightarrow$ zero overlap

12.(C) Only d_{z^2} orbital is aligned along z-axis p_x and p_y orbitals are perpendicular to z-axis d_{zx} orbital lies in the xz plane

SECTION-2

1.(5)

2.(2) $\Delta T_f = K_f \times m$

For KCN solution: $0.80 = K_f \times 0.2 \times 2$ … (1)

$$\begin{array}{cccc} Hg(CN)_2 & + \quad mCN^- & \rightleftharpoons & Hg(CN)_{m+2}^{m-} \\ 0.1 \text{ mole} & 0.2 \text{ mole} & & 0 \\ 0 & (0.2 - 0.1m) \text{ mole} & & 0.1 \text{ mole} \end{array}$$

Final effective molality $= 0.2 - 0.1\,m + 0.1 + 0.2 = 0.5 - 0.1\,m$

Now, $0.60 = K_f \times (0.5 - 0.1\,m)$ … (2)

From (1) and (2) we get

$m = 2$

3.(1) Total charge on four oxide ions = –2 × 4 = –8

Let 'n' number of $X^{-8/3}$ ions will replace O^{2-} ions

$\therefore \quad n \times \frac{-8}{3} = -8 \quad \Rightarrow \quad n = 3$

$\therefore$ One vacancy will be created

4.(79) $6477 \times 10^3 = \frac{4 \times (108 + M_X)}{6 \times 10^{23} \times (577 \times 10^{-12})^3} \quad \Rightarrow \quad M_X = 79$

5.(5) $XeF_2, CO_2, I_3^-, NO_2^+, N_3^-$ are linear with zero dipole-moment.

6.(4) Volume of the unit cell $= \left(408 \times 10^{-10} \text{cm}\right)^3$

$= 67.92 \times 10^{-24} \text{cm}^3$

Mass of the unit cell = d × v

$= 10.6 \times 67.92 \times 10^{-24}$

$= 7.20 \times 10^{-22} \text{g}$

Mass of the unit cell = Number of atoms in unit cell × Mass of each atom

$7.2 \times 10^{-22} = \frac{108}{6.022 \times 10^{23}} \times n \quad \Rightarrow \quad n = 4$

MATHEMATICS

SECTION-1

1.(B) We observe that

$f(9) = f(4+5) = f(4.5) = f(16+4)$

$= f(16 \cdot 4) = f(64) = f(8 \cdot 8)$

$= f(8+8) = f(16) = f(4 \cdot 4) = (4+4) = f(8) = 9$

2.(A) $x \neq (2n+1)\frac{\pi}{2}, n\pi, n \in I.$ The given inequality can be written as

$$\frac{\log_2(x^2 - 8x + 23)}{\log_2 |\sin x|} > \frac{3}{\log_2 |\sin x|}$$

As $\log_2 |\sin x| < 0,$ we get $\log_2(x^2 - 8x + 23) < 3$

$\Rightarrow \quad x^2 - 8x + 23 < 2^3 = 8$

$\Rightarrow \quad x^2 - 8x + 15 < 0$

$\Rightarrow \quad (x-5)(x-3) < 0 \quad \Rightarrow \quad 3 < x < 5$

For $x \in (3, 5), x \neq \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

Hence, $x \in (3, \pi) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 5\right)$

3.(B) $f(x) = f(4-x), f(x) = f(x+10), f(x) = f(14-x).$ There are 201 multiples of 10 and 200 numbers that give remainder 4 when divided by 10.

4.(C) For x-intercept $y = 0$

$\therefore \quad \left||x-2| - a\right| = 3$

$\Rightarrow \quad |x-2| - a = 3$ or $-3 \quad \Rightarrow \quad |x-2| = a+3$ or $a-3$

For 3 x-intercepts

$a + 3 > 0$ and $a - 3 = 0 \quad \ldots (1)$

or $\quad a + 3 = 0$ and $a - 3 > 0 \quad \ldots (2)$

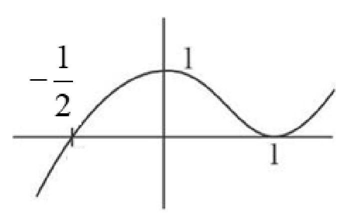
From equation (1) $a = 3$ and equation (2) is rejected

Hence, sum = 3

5.(D) Roots of $f(x) = 0$ are $-\frac{1}{2}$ and 1

Then $f(f(x)) = 0$ (where $f(x) = t$)

$f(t) = 0 \Rightarrow t = -\frac{1}{2}, 1$



So, total number of distinct real solutions = 2 + 1 = 3

6.(BD) Here $E=\sum_{r=0}^{100} {}^{100}C_r\left(x-3\right)^{100-r}\cdot 4^r=\left[(x-3)+4\right]^{100}=(1+x)^{100}$

$\therefore$ Coefficient of x^2 in $E={}^{100}C_2=4950$

$$A_k=\frac{{}^nC_k}{{}^{n+k}C_{k+1}}=\frac{k+1}{n+1}\Rightarrow \sum_{k=0}^{n-1}A_k=\frac{1}{(n+1)}\sum_{k=0}^{n-1}(k+1)$$

$$=\frac{1}{(n+1)}\cdot\frac{n(n+1)}{2}=\frac{n}{2} \qquad ...(i)$$

But given $\sqrt[3]{\sum_{k=0}^{n-1}A_k}=4$

$$\Rightarrow \sum_{k=0}^{n-1}A_k=64 \qquad ...(ii)$$

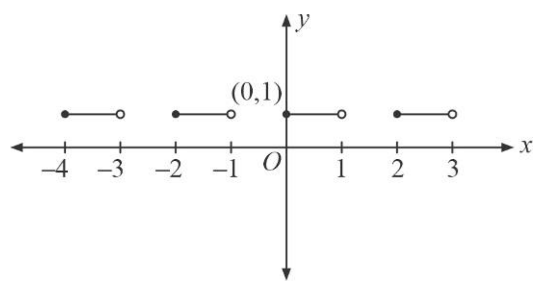
$\therefore$ From (i) and (ii), $\frac{n}{2}=64\Rightarrow 128$

7.(ABCD)

Clearly domain of $f(x)=\bigcup_{n\in I}[2n,2n+1)$

$\Rightarrow$ $f(x)=1, \forall x\in D_f$

Graph of $f(x)=\text{sgn}(\cot^{-1}x)+\tan\frac{\pi}{2}[x]$



From graph $f(x)$ is periodic with period 2

$\Rightarrow$ Option(s) (A), (B), (C) and (D) are correct

8.(ABC)

If $a=3, \sin 9x=\cos 3x, x\in[0,2\pi), 18$ solutions

If $a=2, \sin 4x=\cos 2x, x\in[0,2\pi), 8$ solutions

If $a=1, \sin x=\cos x, x\in[0,2\pi), 2$ solutions

If $a=-1, \sin x=\cos x, x\in[0,2\pi), 2$ solutions

9.(AD) $ax^2+2bx+5c=0$

$D\geq 0$

$4b^2-20ac\geq 0 \qquad \because \qquad b=\frac{a+c}{2}$

$$\left(\frac{a+c}{2}\right)^2-5ac\geq 0$$

$$\Rightarrow a^2+c^2-18ac\geq 0$$

$$\frac{a}{c}+\frac{c}{a}-2\ge 16$$

$$\left(\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}\right)^2\ge 16$$

$$\left|\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}\right|\ge 4$$

10.(AC) Hence $x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}>0\ \ \forall\, x\in R$

Given $(x^2-x+1)^{x-1}<1$

Taking log both sides

$(x-1)\log_{10}(x^2-x+1)<0$

Case I:

$x-1>0, \log_{10}(x^2-x+1)<0$

i.e. $x-1>0$ and $0<x<1\Rightarrow$ no solution

Case II:

$x-1<0$ and $\log_{10}(x^2-x+1)>0$

$\Rightarrow\quad x<1$ and $x>1$ or $x<0\quad\Rightarrow\quad x<0$

OR

Consider a^x where $a>0$

$a^x<1$

(i) When $a>1$ and $x<0$

OR

(ii) When $0<a<1$ and $x>0$

PARAGRAPH FOR Q-11 & 12

11.(B) $x+y=4-z, x^2+y^2=6-z^2$

$\therefore\quad 2xy=(x+y)^2-(x^2+y^2)=(4-z)^2-(6-z^2)=2z^2-8z+10$

Quadratic equation whose roots are x and y is

$t^2-(x+y)t+xy=0$

$t^2-(4-z)t+z^2-4z+5=0$

$D\ge 0$

$(4-z)^2-4(z^2-4z+5)\ge 0$

$\Rightarrow\quad (3z-2)(z-2)\le 0$

$z \in \left[\frac{2}{3}, 2\right]$

$p = \frac{2}{3}, q = 2$

$$f(x) = \frac{4x(x^2+1)}{x^2+(x^2+1)^2} = \frac{4\left(x+\frac{1}{x}\right)}{1+\left(x+\frac{1}{x}\right)^2}$$

Let $\quad x + \frac{1}{x} = t$

$y = \frac{4t}{1+t^2} = \frac{4}{\frac{1}{t}+t}$

For, $\quad x > 0, t \geq 2, t + \frac{1}{t} \geq \frac{5}{2}$

$y \in \left[0, \frac{8}{5}\right]$

12.(B) For exactly one root to lie in $\left(\frac{2}{3}, 2\right)$

$f(2) \cdot f\left(\frac{2}{3}\right) < 0 \qquad \Rightarrow \qquad a \in \left(\frac{1}{3}, 3\right)$

SECTION-2

1.(9) The given expression is equivalent to

$$\frac{f(x+y)}{x+y} - \frac{f(x-y)}{x-y} = 4xy = (x+y)^2 - (x-y)^2$$

$$\Rightarrow \quad \frac{f(x+y)}{x+y} - (x+y)^2 = \frac{f(x-y)}{x-y} - (x-y)^2 = c \text{ (constant)}$$

$$\Rightarrow \quad \frac{f(x)}{x} - x^2 = c \qquad \therefore \qquad f(x) = x^3 + cx \qquad \because \qquad f(1) = -2$$

$$\Rightarrow \quad c = -3 \qquad \therefore \qquad f(x) = x^3 - 3x \qquad \therefore \qquad f(x) + f(-x) = 0$$

$f(-5) + f(-4) + f(0) + f(2) + f(3) + f(4) + f(5)$

$= f(0) + f(2) + f(3)$

$= 0 + 8 - 6 + 27 - 9 = 20$

2.(0) $D_{f_1} = \left[-\frac{1}{2}, 0\right) \cup \left\{\frac{1}{2}\right\}$

$D_{f_2} = [-3, -2]$

3.(8) $g(f(x)) = x$

$f'(x) = \cos(x-1) + 0 + 3x^2 + 4$

$g'(f(x)) \cdot f'(x) = 1$

Put, $x = 1$

$g'(5) \cdot f'(1) = 1$

$$g'(5) = \frac{1}{1+3+4} = \frac{1}{8}$$

4.(7) Put $7\theta = 2n\pi, n \in z$

$\Rightarrow$ $4\theta = 2n\pi - 3\theta$

$\Rightarrow$ $\sin 4\theta = -\sin 3\theta$

$4\sin\theta\cos\theta\,(1 - 2\sin^2\theta) = -3\sin\theta + 4\sin^3\theta$

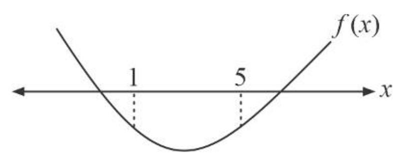
$\Rightarrow$ $16(1-\sin^2\theta)(1 - 4\sin^2\theta + 4\sin^4\theta) = 16\sin^4\theta - 24\sin^2\theta + 9$

$$64\sin^6\theta - 112\sin^4\theta - 56\sin^2\theta - 7 = 0 \begin{cases} \sin^2 2\pi/7 \\ \sin^2 4\pi/7 \\ \sin^2 8\pi/7 \end{cases}$$

Hence, $\sin^2\dfrac{2\pi}{7} + \sin^2\dfrac{4\pi}{7} + \sin^2\dfrac{8\pi}{7} = \dfrac{112}{64} = \dfrac{7}{4}$

5.(5) We have $1 + \sum_{r=1}^{10}(3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r)$

$= 1 + \sum_{r=1}^{10} 3^r \cdot {}^{10}C_r + 10\sum_{r=1}^{10} {}^{9}C_{r-1}$



$= 1 + 4^{10} - 1 + 10 \cdot 2^9$

$= 4^{10} + 5 \cdot 2^{10} = 2^{10}(4^5 + 5) = 2^{10}(\alpha \cdot 4^5 + \beta),$

So, $\alpha = 1$ and $\beta = 5$

Now, $f(1) < 0$ and $f(5) < 0$

$\Rightarrow$ $k \neq 0$

and $f(5) < 0$ $\Rightarrow$ $16 - k^2 < 0$

$\Rightarrow$ $k^2 - 16 > 0$ $\Rightarrow$ $k \in (-\infty, 4) \cup (4, \infty)$

Hence smallest positive integral value of $k = 5$

6.(7) f is an odd function

So, $f(7) = -7, f(5) = 5, f(2) = 3$

$x \to -\infty, f(x) \to x$

$x \to \infty, f(x) \to \infty$